Philadelphia University **Real Analysis 2**
Faculty of Science **Second Quiz**
Department of Basic Sciences and Mathematics
Student name: ___________________________ **Number:_________________**

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1) Determine which of the following functions is Riemann integrable on $[0,1]$. Justify
   a) $f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

   b) $f(x) = \begin{cases} \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

2) Give an example of a function $f : [a, b] \to \mathbb{R}$
   a) That is in $\mathcal{R}[c, b]$ for every $c \in (a, b)$ but which is not in $\mathcal{R}[a, b]$

   b) Such that $f(x) \geq 0 \ \forall \ x \in [a, b]$, $\int_a^b f = 0$, while $f(x) \neq 0$

3) If $f \in \mathcal{R}[a, b]$, prove that $|f| \in \mathcal{R}[a, b]$.

4) Suppose that f is continuous on $[a, b]$, prove that $\exists\, c \in [a, b] \ni \int_a^b f = f(c)(b-a)$.

5) Suppose that f is a bounded function on $[a, b]$ such that either $f^+ = max\{f, 0\}$ or $f^- = min\{f, 0\}$ (but not necessarily both) is in $\mathcal{R}[a, b]$.
Does it necessarily follow that $f \in \mathcal{R}[a, b]$? If so prove it; otherwise provide a counterexample.